[illegible]

Calculus Worksheet: Even and Odd Functions (3)

1. f is an even function and g is an odd function.

   Show that:

   a) $(f \cdot g)(x)$ is an odd function.

   f even $\Rightarrow f(-x) = f(x)$; g odd $\Rightarrow g(-x) = -g(x)$

   Let $h(x) = (f \cdot g)(x) = f(x) \cdot g(x)$

   $h(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-) g(x) = -f(x) \cdot g(x)$

   $h(-x) = -h(x)$ $\quad h(x) = (f \cdot g)(x)$ is odd

   b) $(f / g)(x)$ is an odd function.

   Let $h(x) = (f/g)(x) = \frac{f(x)}{g(x)}$

   $h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -h(x)$

   $\Rightarrow h(x) = (f/g)(x)$ is odd

2. Show that $f(|x|)$ is an even function.

Let $h(x) = f(|x|)$.

$h(-x) = f(|-x|) = f(|x|)$ since $|-x| = |x|$

$\Rightarrow h(x) = h(-x) \Rightarrow h(x) = f(|x|)$ is even.