

Math 125 C, D  
 Winter 2003  
 Midterm I Solutions

1. (a)  $v(t) = \int a(t) dt = \int (-2 \sin t) dt = 2 \cos t + C$  (by guess & check)

$1 = v(0) = 2 \cos(0) + C = 2 + C \Rightarrow C = -1 \Rightarrow v(t) = 2 \cos t - 1$

(b)  $v(t) = 2(\cos t - \frac{1}{2}) \geq 0$  when  $\cos t \geq \frac{1}{2} \Leftrightarrow 0 \leq t \leq \frac{\pi}{3}$  (for  $0 \leq t \leq \pi$ )

So  $|v(t)| = \begin{cases} v(t) = 2 \cos t - 1 & \text{for } 0 \leq t \leq \frac{\pi}{3} \\ -v(t) = 1 - 2 \cos t & \text{for } \frac{\pi}{3} \leq t \leq \pi \end{cases}$

Tot. dist. =  $\int_0^{\pi} |v(t)| dt = \int_0^{\pi/3} (2 \cos t - 1) dt + \int_{\pi/3}^{\pi} (1 - 2 \cos t) dt$   
 $= (2 \sin t - t)|_0^{\pi/3} + (t - 2 \sin t)|_{\pi/3}^{\pi} = (\sqrt{3} - \frac{\pi}{3}) - 0 + (\pi - 0) - (\frac{\pi}{3} - \sqrt{3}) = 2\sqrt{3} + \frac{\pi}{3}$

2. (a)  $\int x e^{x^2} \sec^2(e^{x^2}) dx = \int (\sec^2 u) \cdot \frac{1}{2} du = \frac{1}{2} \tan u + C = \frac{1}{2} \tan(e^{x^2}) + C$

(b)  $\int_1^{\sqrt{e}} \frac{\cos(\pi \ln x)}{x} dx = \int_0^{1/2} \cos(\pi u) du = \frac{1}{\pi} \sin(\pi u)|_0^{1/2} = \frac{1}{\pi} (1 - 0) = \frac{1}{\pi}$

$u = \ln x \quad x = \sqrt{e} \Leftrightarrow u = \ln \sqrt{e} = \frac{1}{2}$   
 $du = \frac{1}{x} dx \quad x = 1 \Leftrightarrow u = \ln 1 = 0$

3. (a) Intersections:  $x = 2\sqrt{x} \Leftrightarrow x^2 = 4x \Leftrightarrow x^2 - 4x = 0 \Leftrightarrow x(x-4) = 0 \Leftrightarrow x = 0, 4$

$A = \int_0^4 (f_2(x) - f_1(x)) dx = \int_0^4 (2\sqrt{x} - x) dx$   
 $= (\frac{4}{3} x^{3/2} - \frac{1}{2} x^2)|_0^4 = (\frac{4}{3} \cdot 8 - \frac{1}{2} \cdot 16) - (0) = \frac{8}{3}$

(b) Shells. Volume of shell near  $x_i \approx 2\pi r h \Delta x \approx 2\pi (x_i + 1)(2\sqrt{x_i} - x_i) \Delta x$

Volume  $\approx \sum_{0 \leq x_i \leq 4} 2\pi (x_i + 1)(2\sqrt{x_i} - x_i) \Delta x$

Volume =  $\int_0^4 2\pi (x+1)(2\sqrt{x} - x) dx$

4. (a)  $V = \int_a^b \pi (f(x))^2 dx = \int_0^1 \pi (\sqrt{\sin^{-1} x})^2 dx = \int_0^1 \pi \sin^{-1} x dx$

Solve for  $x$ :  $y^2 = \sin^{-1} x, x = \sin(y^2)$ . Upper limit:  $y = \sqrt{\sin^{-1}(1)} = \sqrt{\frac{\pi}{2}}$

$V = \int_0^{\sqrt{\pi/2}} 2\pi y (g_2(y) - g_1(y)) dy = \int_0^{\sqrt{\pi/2}} 2\pi y (1 - \sin(y^2)) dy$   
 $= \int_0^{\pi/2} \pi (1 - \sin u) du$  [ $u = y^2, y = \sqrt{u} \Leftrightarrow u = \pi/2$   
 $du = 2y dy, y = 0 \Leftrightarrow u = 0$ ]  
 $= \pi (u + \cos u)|_0^{\pi/2} = \pi [(\frac{\pi}{2} + 0) - (0 + 1)] = \frac{\pi^2}{2} - \pi$

5. (a)  $s(\frac{1}{2}) = \int_0^{\pi/2} \sqrt{4 \sin^2 x + 1} dx$ . Let  $a=0, b=\frac{\pi}{2}, n=3, \Delta x = \frac{b-a}{n} = \frac{\pi}{6}; x_0=0, x_1=\frac{\pi}{6}, x_2=\frac{\pi}{3}, x_3=\frac{\pi}{2}$

$R_3 = \Delta x (f(x_1) + f(x_2) + f(x_3)) = \frac{\pi}{6} (\sqrt{4 \sin^2 \frac{\pi}{6} + 1} + \sqrt{4 \sin^2 \frac{\pi}{3} + 1} + \sqrt{4 \sin^2 \frac{\pi}{2} + 1})$   
 $= \frac{\pi}{6} (\sqrt{2} + 2 + \sqrt{5}) \approx 2.95848$

(b) Let  $G(u) = \int_0^u \sqrt{4 \sin^2 x + 1} dx$ . Then  $G'(u) = \sqrt{4 \sin^2 u + 1}$  (by F.T.C. Part 1)

Since  $s(t) = G(\pi t), v(t) = \frac{ds}{dt} = G'(\pi t) \cdot \pi = \pi \sqrt{4 \sin^2(\pi t) + 1}$