Solving Trigonometric Equations

We solve trigonometric equations just as we solve any other equation. We isolate the thing that we are trying to evaluate and then figure out what its value is. The difference is that with trigonometric equations, we first isolate an expression such as sin x or cos x, and then we evaluate these trig expressions for x.

Example 1: Solve: $\sin x \cos x - \sin x = 0$ in the interval $[0, 2\pi)$.

Solution:

$$\sin x \cos x - \sin x = 0$$
$$(\sin x)(\cos x - 1) = 0$$
$$\therefore \sin x = 0 \text{ or } \cos x - 1 = 0$$

$$\sin x = 0 \qquad\qquad \cos x - 1 = 0$$
$$x = 0 \text{ or } \pi \qquad\qquad \cos x = 1$$
$$x = 0$$

So the solutions are 0 and π.

Example 2: Solve: $\cos 2x + 3 \sin x - 2 = 0$ in the interval $[0, 2\pi)$.

Solution:

$$\cos 2x + 3 \sin x - 2 = 0$$
$$(1 - 2 \sin^2 x) + 3 \sin x - 2 = 0$$
$$-2 \sin x^2 + 3 \sin x - 1 = 0$$

If we do a change of variable at this point, replacing "sin x" with "u", we see that this is in the form of a quadratic equation:

$$-2u^2 + 3u - 1 = 0$$
$$(-2u + 1)(u - 1) = 0$$

So,

$$-2 \sin x^2 + 3 \sin x - 1 = 0$$
$$(-2 \sin x + 1)(\sin x - 1) = 0$$
$$\therefore -2 \sin x + 1 = 0 \text{ or } \sin x - 1 = 0$$

$$-2 \sin x + 1 = 0 \qquad\qquad \cos x - 1 = 0$$
$$\sin x = \tfrac{1}{2} \qquad\qquad \sin x = 1$$
$$x = \tfrac{\pi}{6} \text{ or } \tfrac{5\pi}{6} \qquad\qquad x = \tfrac{\pi}{2}$$

So the solutions are $\frac{\pi}{6}$, $\frac{\pi}{2}$ and $\frac{5\pi}{6}$.

Example 3: Solve: $\sin 2x = 0$ in the interval $[0, 2\pi)$.

Solution:

$$\sin 2x = 0$$
$$\sin \theta = 0 \rightarrow \theta = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$
$$x = 0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}, 2\pi, \ldots$$

So the values for x within the interval are $x = 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$.

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