Conic Sections:

The Parabola

In earlier courses, you've looked at the parabola as the set of solutions to a quadratic equation. Now, you will start to see it as one of the **conic sections**. You'll learn about the centre of the parabola, the **focus**, and a line that has special properties related to the focus, the **directrix**.

FORMULAS FOR PARABOLAS

When the vertex of the parabola is at the origin, the formula is simple. For a parabola with a vertical line of symmetry (one that opens up or down), x is squared. For a parabola with a horizontal line of symmetry (one that opens right or left), y is squared.

Standard form:	$x^2 = 4py$	$y^2 = 4px$
Vertex:	(0, 0)	(0, 0)
Focus:	(0, p)	(p, 0)
Directrix:	$y = -p$	$x = -p$

When the vertex is not at origin, then the parabola has been translated (it has undergone a shift or a slide).

Standard form:	$(x - h)^2 = 4p(y - k)$	$(y - k)^2 = 4p(x - h)$
Vertex:	(h, k)	(h, k)
Focus:	(h, k + p)	(h + p, k)
Directrix:	$y = k - p$	$x = h - p$

Example 1: Determine the vertex, focus and directrix of the parabola $x^2 + 8x + 8y = 8$.

Solution: First we need to get the parabola into the standard form. We can tell that the parabola is a vertical parabola, since there is an x^2 term instead of a y^2 term. To convert our equation to standard form, we start by completing the square.

$$\begin{aligned} x^2 + 8x + 8y &= 8 \\ x^2 + 8x &= 8 - 8y \\ x^2 + 8x + \left(\tfrac{8}{2}\right)^2 &= 8 - 8y + \left(\tfrac{8}{2}\right)^2 \\ x^2 + 8x + 16 &= 24 - 8y \\ (x + 4)^2 &= 24 - 8y \end{aligned}$$

Now we need to format the right-hand side so that it matches standard form. We factor out 4p from both terms:

$$\begin{aligned} (x + 4)^2 &= -8(y - 3) \\ &= 4 \cdot (-2)(y - 3) \\ \therefore p = -2, h &= -4, k = 3 \end{aligned}$$

The vertex is at (−4, 3), the focus is at (−4, 3 − 2) = (−4, 1) and the directrix is $y = 3 - (-2)$ or $y = 5$.

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