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CAPE Mathematics Unit 2
Worksheet 4: Sequences and Series

1. (i) Prove by mathematical induction that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ for $n \in \mathbf{Z}^+$.

(ii) Hence or otherwise, find in terms of n the sum of the cubes of the first n even positive integers.

2. (i) An arithmetic series has nth term a + (n − 1)d where a and d are real constants. Prove that the nth partial sum S_n is given by $S_n = \frac{n}{2}(2a+(n-1)d)$.

(ii) Hence express $\ln(2\times 2^2 \times 2^3 \times ... \times 2^{48} \times 2^{49})$ in the form $k\ln 2$ where k is an integer.

3. Express the infinite series 3 + 5 + 7 + 9 + ... in sigma notation.

4. Find the sum of the convergent geometric series $S = 2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ...$

5. (i) Given that S_r denotes the sum of the first r positive integers, prove by mathematical induction that $S_r = \frac{1}{2}r(r+1)$.

(ii) Hence, find the sum $S = \sum_{r=1}^{n} S_r$ in terms of n.

6. The common ratio of a geometric series is given by $r = \frac{5x}{4+x^2}$. Find all the values of x for which the geometric series converges.

7. A sequence $\{u_n\}$ of real numbers satisfies $u_{n+1}u_n = 3(-1)^n$; $u_1 = 1$.

(a) Show that

(i) $u_{n+2} = -u_n$

(ii) $u_{n+4} = u_n$

(b) Write the FIRST FOUR terms of this sequence.

8. Verify that the sum, S_n, of the series $\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + ...$, to n terms, is $S_n = \frac{2}{3}\left(1 - \frac{1}{2^{2n}}\right)$.

9. Three consecutive terms, $x-d$, x and $x+d$, $d>0$, of an arithmetic series have sum 21 and product 315. Find the value of

(i) x

(ii) the common difference d.

10. Three sequences are given below.

1, 4, 7, 10, ...

$1, -\frac{1}{4}, \frac{1}{7}, -\frac{1}{10}, ...$

$(-1)^1, (-1)^4, (-1)^7, (-1)^{10}, ...$

Determine which of the sequences is convergent, divergent or periodic and state which of the sequences is an arithmetic sequence.

11. (i) Show that the series

$\log_a b + \log_a(bc) + \log_a(bc^2) + ... + \log_a(bc^{n-1})$,

where a, b, c > 0, $n \geq 1$

is an arithmetic progression whose sum, S_n, to n terms is $\frac{n}{2}\log_a b^2c^{n-1}$.

(ii) Find S_n when n = 6 and a = b = c = 5.