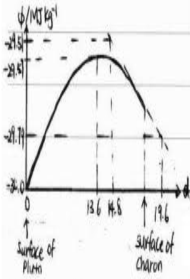

The graph shown on the right is the ϕ-d graph of two planets, Pluto and Charon

a) All values of gravitational potential are negative as it is work done per unit mass by an external force to bring a point mass from infinity to that point. Since work is done by the attractive gravitational force, work done by an external force is negative

b) Gradient at a point on the ϕ-d graph gives magnitude of the acceleration of free fall at that point as the acceleration of free fall at a point is equivalent to the gravitational field strength at that point and gravitational field strength is proportional to the gravitational potential gradient at that point.

c) Determine the distance from the surface of Pluto at which the acceleration of free fall is zero from ϕ-d graph.

Gradient of ϕ-d graph is zero at $d=13.6\times10^6$ m.

Since magnitude of acceleration of free fall is equal to the potential gradient, it is zero at $d=13.6\times10^6$ m

d) Determine the acceleration of free fall on the surface of Charon is equal to the gradient of tangent at Charon's surface.

From the graph, gradient of tangent $= \frac{(-29.79-(-29.51))\times10^6}{(19.6-14.8)\times10^6} = -5.83\times10^{-2}$ m s^{-2}

Hence, acceleration of free fall is 5.83×10^{-2} m s^{-2} towards Charon.

e) A lump of rock of mass 2.5kg is ejected from the surface of Charon such that it travels towards Pluto. Using data from the graph, determine the minimum speed with which the rock hits the surface of Pluto.

Assuming no loss of energy in the rock's travel to Pluto, sufficient energy has to be provided such that the rock gets beyond Charon's gravitation attraction which is when $d=13.6\times10^6$ m from the Pluto's surface.

Assume that the rock is just able to escape Charon's gravitational pull,

Total energy at $d=13.6\times10^6$ m = Total energy at Pluto's surface

(KE + GPE) at $d=13.6\times10^6$ m = (KE + GPE) at Pluto's surface

$$0 + m\phi_{d=13.6\times10^6\,m} = \frac{1}{2}mv^2 + m\phi_{\text{Pluto's surface}}$$

$$v = \sqrt{2\times(-29.57-(-30.0))\times10^6}$$

$\therefore$ Minimum speed $v = 9.3\times10^2$ m s^{-1}

Other possible assumptions:

1. No air resistance (work done against air resistance is negligible)
2. The movement of the planets are negligible throughout the flight.
3. Effects of gravitational field set up by other heavenly bodies are negligible.

f) When the rock travels from Pluto to Charon, minimum speed on reaching Charon is different from that calculated in (e). Explain why.

The speed gained when the rock travels is dependent on the loss in GPE between its start and end point. However, the minimum speed attained assumes that the rock just comes to rest at the point where gravitational potential is maximum which is the equilibrium position between Pluto and Charon. Since the loss in GPE in moving from equilibrium position to Charon is less than to Pluto, the minimum speed on reaching Charon is lower.

i) Given $R_E = 6400$ km, find ϕ.

$$\phi = -\frac{GM}{R_E}$$
$$= -\frac{GM}{R_E^2}R_E$$
$$= -gR_E$$
$$= -9.81\times6400\times10^3$$
$$= -6.28\times10^7 \text{ J kg}^{-1}$$

ii) Find escape speed of an object from the Earth.

Gain in KE = Loss in GPE

$$\frac{1}{2}mv^2 = 0-\left(-\frac{GMm}{R_E}\right)$$
$$v = \sqrt{\frac{2GM}{R_E}}$$
$$= \sqrt{\frac{2GM}{R_E^2}R_E}$$
$$= \sqrt{2gR_E}$$
$$= \sqrt{2\times9.81\times6400\times10^3}$$
$$= 1.12\times10^4 \text{ m s}^{-1}$$

iii) In practice, the escape speed can have a value less than that calculated in (ii)

The object, while on the Earth, already possess kinetic energy by virtue of the Earth's rotation. Hence less additional energy will be needed to escape from the Earth so escape speed of the object can be lower.

iv) Express escape speed v in terms of ρ, G and R.

$$V = \sqrt{\frac{2GM}{R}}$$
$$= \sqrt{\frac{2G\rho V}{R}}$$
$$= \sqrt{\frac{2G\rho(\frac{4}{3}\pi R^3)}{R}}$$
$$= R\sqrt{\frac{8\pi G\rho}{3}}$$

Note: escape speed is the initial speed required to propel an object so that it can escape from the gravitational field of a planet and travel to infinity.

v) For black holes, the magnitude of escape velocity may be greater than the speed of light. Derive an expression for critical radius of a star at which it will behave as a black hole.

$$c = R\sqrt{\frac{8\pi G\rho}{3}}$$

Critical radius, $R = c\sqrt{\frac{3}{8\pi G\rho}}$