$\sin^2 x + \cos^2 x = 1$ $\quad \sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\quad \cos 2x = \cos^2 x - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ $\quad \cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\quad \cos 2x = 2\cos^2 x - 1$

$1 + \cot^2 x = \csc^2 x$ $\quad \sin 2x = 2\sin x \cos x$ $\quad \cos 2x = 1 - 2\sin^2 x$

FACTORING POLYNOMIALS

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

PROPERTIES OF EXPONENTIAL FUNCTIONS

$a^x a^y = a^{x+y}$ $\quad (a^x)^y = a^{xy}$ $\quad (ab)^x = a^x b^x$

$\frac{a^x}{a^y} = a^{x-y}$ $\quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ $\quad \sqrt[n]{x} = (x)^{\frac{1}{n}}$

TRIGONOMETRY

Deg	Rad	SIN	COS	TAN
30	$\pi/6$	0.5	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60	$\pi/3$	$\sqrt{3}/2$	0.5	$\sqrt{3}$
90	$\pi/2$	1	0	∞
120	$2\pi/3$	$\sqrt{3}/2$	-0.5	$-\sqrt{3}$
135	$3\pi/4$	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1
150	$5\pi/6$	0.5	$-\sqrt{3}/2$	$-1/\sqrt{3}$
180	π	0	-1	0
210	$7\pi/6$	-0.5	$-\sqrt{3}/2$	$1/\sqrt{3}$
225	$5\pi/4$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1
240	$4\pi/3$	$-\sqrt{3}/2$	-0.5	$\sqrt{3}$
270	$3\pi/2$	-1	0	$-\infty$
300	$5\pi/3$	$-\sqrt{3}/2$	0.5	$-\sqrt{3}$
315	$7\pi/4$	$-1/\sqrt{2}$	$1/\sqrt{2}$	-1
330	$11\pi/6$	-0.5	$\sqrt{3}/2$	$-1/\sqrt{3}$
0,360	2π	0	1	0

FUNDAMENTAL THEOREM OF CALCULUS

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

$\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative of f $F'(x) = f(x)$

SUBSTITUTION RULE FOR DEF. INTEGRALS

$\int_a^b f(g(x))\cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\cdot u'$ where $u = g(x)$ Δ limits of Integ.

VOLUMES OF SOLIDS

dy $f(x)$ = radius, r = radius, h = height

Disc Method: $V = \int_a^b \pi (f(x))^2\,dx$

Disc W/ Hole: $V = \int_a^b \pi \{(f(x))^2 - (g(x))^2\}\,dx$

Shell Method: $V = 2\pi \int_a^b r h\,dx$

Cross Section: $V = \int_a^b A(x)\,dx$

DISC	SHELL
$f(x) - x$	$f(x) - y$
$g(y) - y$	$g(y) - x$

INTEGRATION FORMULAS - Assume +C For Indefinite Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1}$ $\quad \int u^n \cdot u' = \frac{u^{n+1}}{n+1}$ $\quad \int a^x dx = \frac{a^x}{\ln a}$ $\quad \int e^u \cdot u' = e^u$ $\quad \int \frac{1}{x} dx = \ln|x|$ $\quad \int \frac{u'}{u} = \ln|u|$

$\int \sin u \cdot u' = -\cos u$ $\quad \int \cos u \cdot u' = \sin u$ $\quad \int \sec^2 u \cdot u' = \tan u$ $\quad \int \csc^2 u \cdot u' = -\cot u$

$\int \tan u \cdot u' = -\ln|\cos u|$ $\quad \int \sec u \tan u \cdot u' = \sec u$ $\quad \int \sec u \cdot u' = \ln|\sec u + \tan u|$

$\int \cot u \cdot u' = \ln|\sin u|$ $\quad \int \csc u \cot u \cdot u' = -\csc u$ $\quad \int \csc u \cdot u' = \ln|\csc u - \cot u|$

$\int \sinh x = \cosh x$ $\quad \int \cosh x = \sinh x$ $\quad \int \sec^3 u \cdot u' = \frac{1}{2}(\sec u \tan u + \ln|\sec u + \tan u|)$

$\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ $\quad \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right)$ $\quad \int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$ $\quad \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

INTEGRATION BY PARTS

$\int u \cdot v' = uv - \int v \cdot u'$ $\quad u= \quad v'= \quad u'= \quad v=$

- Pick a u that will simplify the rt. side of the eq. as u'
- Pick a v' that is the most complex - if you can integrate it

TRIGONOMETRIC INTEGRALS

For cases of ① $\int \sin^m x \cos^n x$ and ② $\int \tan^m x \sec^n x$ → $\int u^n u' = \frac{u^{n+1}}{n+1}$

① • If n is odd, save one $(\cos x)$ and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine. Get it into the form of one or more $\boxed{\int \sin^n x \cos x\,dx = \sin^{n+1} x / n+1 + C}$

• If m is odd, save one $(\sin x)$ and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine. Get it into the form of one or more $\boxed{\int \cos^n x \sin x\,dx = -\cos^{n+1} x / n+1 + C}$

➡ If both m and n are even, use the half angle identities $\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$ and $\boxed{\cos^2 x = \frac{1}{2}(1+\cos 2x)}$

② • If n is even, save one $(\sec^2 x)$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$. Get it into the form of one or more $\boxed{\int \tan^n x \sec^2 x\,dx = \tan^{n+1} x / n+1 + C}$

• If m is odd, save one $(\sec x \tan x)$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$. Get it into the form of $\boxed{\int \sec^n x \sec x \tan x\,dx = \sec^{n+1} x / n+1 + C}$

➡ It is sometimes helpful to use the identity $\boxed{\sin x \cos x = \frac{1}{2}\sin 2x}$

MORE TRIG INTEGRALS

ⓐ $\int \sin mx \cos nx\,dx$ $\quad A = mx$

ⓑ $\int \sin mx \sin nx\,dx$ $\quad B = nx$

ⓑ $\int \cos mx \cos nx\,dx$

← For each of these cases → use:

ⓐ $\int \sin A \cos B = \frac{1}{2}\int \sin(A-B) + \sin(A+B)$

ⓑ $\int \sin A \sin B = \frac{1}{2}\int \cos(A-B) - \cos(A+B)$

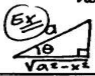
ⓒ $\int \cos A \cos B = \frac{1}{2}\int \cos(A-B) + \cos(A+B)$

TRIGONOMETRIC SUBSTITUTION

		Radical Becomes
$\sqrt{a^2 - x^2}\,dx$	$x = a\sin\theta$ $\quad dx = a\cos\theta\,d\theta$	$a\cos\theta$
$\sqrt{a^2 + x^2}\,dx$	$x = a\tan\theta$ $\quad dx = a\sec^2\theta\,d\theta$	$a\sec\theta$
$\sqrt{x^2 - a^2}\,dx$	$x = a\sec\theta$ $\quad dx = a\sec\theta\tan\theta\,d\theta$	$a\tan\theta$

- Radical can be in the numerator or denominator

Ex: $x = a\sin\theta$ $\quad \sin\theta = \frac{x}{a}$



- Use Triangle to get everything back into terms of x and a

PARTIAL FRACTIONS

Rational Functions $P(x)/Q(x)$ | If Deg $P(x) \geq$ Deg $Q(x)$ - LONG DIVISION | If Deg $Q(x) >$ Deg $P(x)$ - PF : 1st Factorize $Q(x)$ - 4 Possibilities

① Distinct Linear Factors: $Q(x) = (a_1x+b_1)(a_2x+b_2)\cdots(a_kx+b_k)$; $\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_k}{a_kx+b_k}$

② Repeated Linear Factors: $Q(x) = (a_1x+b_1)^r$; $\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{(a_1x+b_1)^2} + \cdots + \frac{A_r}{(a_1x+b_1)^r}$

③ Distinct Irreducible Quadratics: $Q(x) = (ax^2+bx+c)$, $b^2 - 4ac < 0$; $\frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c}$

④ Repeated Irreducible Quadratics: $b^2 - 4ac < 0$; $Q(x) = (ax^2+bx+c)^r$, $b^2-4ac<0$; $\frac{P(x)}{Q(x)} = \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$

- Also - Separate the Fraction - Complete the square etc...

L'Hopitals Rule

For cases of Indeterminate Forms: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ $\quad \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ $\quad 0\cdot\infty, \infty-\infty, 0^0, \infty^0, 1^\infty$

DERIVATIVES OF INVERSE TRIG FUNCTIONS

$D = d/dx$

$D\sin^{-1}x = 1/\sqrt{1-x^2}$ $\quad D\cos^{-1}x = -1/\sqrt{1-x^2}$ $\quad D\tan^{-1}x = 1/1+x^2$

$D\csc^{-1}x = -1/x\sqrt{x^2-1}$ $\quad D\sec^{-1}x = 1/x\sqrt{x^2-1}$ $\quad D\cot^{-1}x = -1/1+x^2$

RATIONALIZING SUBSTITUTIONS

$u = \sqrt[n]{g(x)}$ $\quad$ Eliminate Roots with Least Common Multiple

STRATEGY FOR INTEGRATION

① SIMPLIFY THE INTEGRAND

- Substitute Trig Identities
- ● ALGEBRAIC MANIPULATION
- Separate the Fraction
- Multiply By the Conjugate
- Complete the Square
- Adding and Subtracting
- Multiply by 1/1, a/a, e^x/e^x etc...

② LOOK FOR AN OBVIOUS SUBSTITUTION Find $g(x)$ and $g'(x)$ in the integrand

③ CLASSIFY THE INTEGRAND

- $\boxed{\text{TRIG FUNCTIONS}}$ - Powers, Use Trig. Integrals; Rational Functions use WEIERSTRAUSS Sub.
- $\boxed{\text{RATIONAL FUNCTIONS}}$ - Use Partial Fractions or Long Division First
- $\boxed{\text{PRODUCT OF 2 FUNCTIONS}}$ - Integration By Parts
- $\boxed{\text{RADICALS}}$ $\sqrt{\pm a^2 \pm x^2}$ Trig Sub $\quad \sqrt[n]{ax+b}$ $u = \sqrt[n]{g(x)}$ Rationalizing Sub.

④ USE SEVERAL METHODS - ONLY 2 REAL ONES : PARTS and SUBSTITUTION

⑤ RELATE THE PROBLEM TO PREVIOUS PROBLEMS - TRY AGAIN

WEIERSTRAUSS SUBSTITUTION

For any rational function $f(x)$ of $\sin x$ and $\cos x$

$t = \tan\left(\frac{x}{2}\right) = \frac{\sin x}{1+\cos x}$

$\sin x = \frac{2t}{1+t^2}$ $\quad \cos x = \frac{1-t^2}{1+t^2}$

$dx = \frac{2}{1+t^2}\,dt$

APPROXIMATE INTEGRATION

Riemann Sums with finite n's. Accuracy: $S_n > M_n > T_n > R_n \approx L_n$. Use S_n For Elmin. All Approx. For the def. integral $\int_a^b f(x)\,dx$

$L_n = \frac{b-a}{n}\left(f(x_0) + f(x_1) + \cdots + f(x_{n-1})\right)$

$R_n = \frac{b-a}{n}\left(f(x_1) + f(x_2) + \cdots + f(x_n)\right)$

$M_n = \frac{b-a}{n}\left(f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$

$T_n = \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots 2f(x_{n-1}) + f(x_n)\right)$

$S_n = \frac{b-a}{3n}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)\right)$

$|E_{T_n}| \leq \frac{\max|f''(x)|(b-a)^3}{12n^2}$ $\quad |E_{M_n}| \leq \frac{\max|f''(x)|(b-a)^3}{24n^2}$ $\quad |E_{S_n}| \leq \frac{\max|f^4(x)|(b-a)^5}{180n^4}$

for $f''(x)$ and $f^4(x)$, Max is on $[a,b]$. $|E_{S_n}|$ for a cubic function is always 0

IMPROPER INTEGRALS

■ Convergent: Lim Exists as a Finite #, N ■ Divergent: Lim Doesn't Exist as #, $\pm\infty$

Limits of Integ. @ $\pm\infty$

- If Integral Exists for $t \geq a$, $\int_a^\infty f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$
- If Integral Exists for $t \leq b$, $\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty} \int_t^b f(x)\,dx$

Discontinuities at a or b

- $f(x)$ cont. on $[a,b)$, Disc @ b: $\int_a^b f(x)\,dx = \lim_{t\to b^-} \int_a^t f(x)\,dx$
- $f(x)$ cont. on $(a,b]$, Disc @ a: $\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx$

COMPARISON THEOREM FOR IMPROPER INTEGRALS

If f and g are continuous and $f(x) \geq g(x) \geq 0$

① If $\int_a^\infty f(x)\,dx$ is CONVERGENT, $\int_a^\infty g(x)$ is CONVERGENT

② If $\int_a^\infty g(x)\,dx$ is DIVERGENT, $\int_a^\infty f(x)\,dx$ is DIVERGENT

→ Starting point a doesn't matter

? $\int_a^\infty g(x)$ and $f(x)$ is Divergent, try a function $< g(x)$

? $\int_a^\infty f(x)$ and $g(x)$ is CONVERGENT, try a function $> f(x)$

If both fail, you can't tell (Use L'Hop to do limits)

ARC LENGTH and S.A.

$S = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$

$SA_x = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$

$SA_y = \int_a^b 2\pi x \sqrt{1 + (f'(y))^2}\,dx$ ($x = f(y)$)

SA_x - Ab. x-axis SA_y - Ab. y-axis