Worksheet: Exponential and Logarithmic Functions
Solutions

Identities and Equivalent Equations:
For all of the following the base is a positive real (but not 1): $a > 0$, $a \neq 1$.

- Definition of Logarithm:

$$a^u = v \quad \Leftrightarrow \quad \log_a v = u, \qquad u \in \mathbb{R},\ v > 0 \tag{1}$$

- Equivalent Equations:

$$\log_a u = \log_a v \quad \Leftrightarrow \quad u = v, \qquad u, v > 0 \tag{2}$$
$$a^u = a^v \quad \Leftrightarrow \quad u = v, \qquad u, v \in \mathbb{R} \tag{3}$$

  Remember that rule (2) can create extraneous solutions;
  check your results in the *original* equation.

- Identities:

$$a^{\log_a u} = u, \qquad u > 0 \tag{4}$$
$$\log_a a^u = u, \qquad u \in \mathbb{R} \tag{5}$$
$$\log_a (uv) = \log_a u + \log_a v, \qquad u, v > 0 \tag{6}$$
$$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v, \qquad u, v > 0 \tag{7}$$
$$\log_a u^r = r \log_a u, \qquad u > 0,\ r \in \mathbb{R} \tag{8}$$
$$\log_a u = \frac{\log_b u}{\log_b a}, \qquad u, a, b > 0,\ b \neq 1 \tag{9}$$

Problems:

1. Solve

$$\ln(k+5) + \ln(k+2) = \ln(14k)$$

   Solution:
   Using rules (6) and (2) from the list above, we can convert this logarithmic equation into a polynomial equation which we can then solve for k.

$$\begin{aligned} \ln(k+5) + \ln(k+2) = \ln(14k) \quad &\Longrightarrow \quad \ln\left[(k+5)(k+2)\right] = \ln(14k) \\ &\Longrightarrow \quad (k+5)(k+2) = 14k \\ &\Longrightarrow \quad k^2 + 7k + 10 = 14k \\ &\Longrightarrow \quad k^2 - 7k + 10 = 0 \\ &\Longrightarrow \quad (k-5)(k-2) = 0 \\ &\Longrightarrow \quad k \in \{2, 5\}. \end{aligned}$$