

Mathematical Induction and Proof of Recursion

10/10/2020

Prove Algebra 2 Math Exam Review (MATH202)

PROVE BY INDUCTION 1.1, 1.2, 1.3, 1.4, 1.5, 1.6

Problem 1.1.1

1. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$ 2. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$ 3. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$
4. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$ 5. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$
6. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$ 7. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$
8. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$ 9. $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$

Problem 1.2

10. Prove using induction: $(x^n - 1)^2 = (x - 1)^2 (x^{n-1} + x^{n-2} + \dots + 1)^2$
11. Prove using long division: $(x^n - 1)^2 = (x - 1)^2 (x^{n-1} + x^{n-2} + \dots + 1)^2$

Problem 1.3

12. Prove by induction: $(x^n - 1)^2 = (x - 1)^2 (x^{n-1} + x^{n-2} + \dots + 1)^2$
13. Prove by induction: $(x^n - 1)^2 = (x - 1)^2 (x^{n-1} + x^{n-2} + \dots + 1)^2$

Problem 1.4

14. Prove by induction: $(x^n - 1)^2 = (x - 1)^2 (x^{n-1} + x^{n-2} + \dots + 1)^2$

n	Statement
1	$(x^1 - 1)^2 = (x - 1)^2 (1)^2$
2	$(x^2 - 1)^2 = (x - 1)^2 (x + 1)^2$
3	$(x^3 - 1)^2 = (x - 1)^2 (x^2 + x + 1)^2$
4	$(x^4 - 1)^2 = (x - 1)^2 (x^3 + x^2 + x + 1)^2$
5	$(x^5 - 1)^2 = (x - 1)^2 (x^4 + x^3 + x^2 + x + 1)^2$

Assume: $(x^k - 1)^2 = (x - 1)^2 (x^{k-1} + x^{k-2} + \dots + 1)^2$

Goal: $(x^{k+1} - 1)^2 = (x - 1)^2 (x^k + x^{k-1} + \dots + 1)^2$

15. Prove the sum of 10 terms: $\sum_{i=1}^{10} i^2 = 385$

n	Statement
1	$1^2 = 1$
2	$1^2 + 2^2 = 5$
3	$1^2 + 2^2 + 3^2 = 14$
4	$1^2 + 2^2 + 3^2 + 4^2 = 30$
5	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$
6	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$
7	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = 140$
8	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = 203$
9	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 = 285$
10	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 385$

16. Prove by induction: $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$