

Mathematical Induction and Proof of Recursion

10/20/2023

Discrete Algebra 2 Midterm Review (MATH 250)

PROB 1: PROBLEMS 1.1, 1.2, 1.3, 1.4, 1.5, 1.6

Problem 1.1.1

- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$
- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$
- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$
- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$
- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$
- $$\frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n} = \frac{(2n+1) \cdot 2^n + 2^{n+1} + 2^n}{2^{n+1} + 2^n - 2^n - 2^n}$$

Problem 1.1.2

- $$(x^2 - 1)^2 - (x^2 - 1)(x + 1) = (x^2 - 1)(x - 1) - (x^2 - 1)(x + 1)$$
- $$(x^2 - 1)^2 - (x^2 - 1)(x + 1) = (x^2 - 1)(x - 1) - (x^2 - 1)(x + 1)$$

Problem 1.2

- $$f(x) = -x^2 + 2x - 1 = -(x-1)^2$$
- $$f(x) = -x^2 + 2x - 1 = -(x-1)^2$$

Problem 1.3

1.3.1. Prove by induction that without using the set of bits

n	Sum
1	1
2	3
3	7
4	15
5	31

$$S_n = 2^n - 1$$

1.3.2. Prove the sum of 10 bits is 1023

Position	Value
9	512
8	256
7	128
6	64
5	32
4	16
3	8
2	4
1	2
0	1

1.3.3. Prove the sum of 10 bits is 1023

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$$