3. Prime Numbers

(135) Using computer software, write a program
(a) to generate all Mersenne primes up to $2^{525} - 1$;
(b) to determine the smallest prime number larger than $10^{100} + 1$.

(136) Write a program that generates prime numbers up to a given number N. One can, of course, use Eratosthenes' sieve.

(137) Use a computer to find four consecutive integers having the same number of prime factors (allowing repetitions).

(138) (a) By reversing the digits of the prime number 1009, we obtain the number 9001, which is also prime. Write a program to find the prime numbers in [1,10000] verifying this property.
(b) By reversing the digits of the prime number 163, we obtain the number 361, which is a perfect square. Using computer software, write a program to find all prime numbers in $[1, 10000]$ with this property.

(139) Using a computer, find all prime numbers $p \leq 10\,000$ with the property that p, $p + 2$ and $p + 6$ are all primes.

(140) Let p_k be the k-th prime number. Show that $p_k < 2^k$ if $k \geq 2$.

(141) If a prime number $p_k > 5$ is equally isolated from the prime numbers appearing before and after it, that is $p_k - p_{k-1} = p_{k+1} - p_k = d$, say, show that d is a multiple of 6. Then, for each of the cases $d = 6$, 12 and 18, find, by using a computer, the smallest prime number p_k with this property.

(142) Prove that none of the numbers

$$12321,\ 1234321,\ 123454321,\ 12345654321,\ 1234567654321,$$
$$123456787654321,\ 12345678987654321$$

is prime.

(143) For each integer $k \geq 1$, let n_k be the k-th composite number, so that for instance $n_1 = 4$ and $n_{10} = 18$. Use computer software and an appropriate algorithm in order to establish the value of n_k, with $k = 10^\alpha$, for each integer $\alpha \in [2, 10]$.

(144) For each integer $k \geq 1$, let n_k be the k-th number of the form p^α, where p is prime, α a positive integer, so that for instance $n_1 = 2$ and $n_{10} = 16$. Use computer software and an appropriate algorithm in order to establish the value of n_k, with $k = 10^\alpha$, for each integer $\alpha \in [2, 10]$.

(145) Find all positive integers $n < 100$ such that $2^n + n^2$ is prime. To which class of congruence modulo 6 do these numbers n belong?

(146) Show that if the integer $n \geq 4$ is not an odd multiple of 9, then the corresponding number $a_n := 4^n + 2^n + 1$ is necessarily composite. Then, use a computer in order to find all positive integers $n < 1000$ for which a_n is prime.

(147) Consider the sequence (a_n) defined by $a_1 = a_2 = 1$ and, for $n \geq 3$, by $a_n = n! - (n-1)! + \cdots + (-1)^n 2! + (-1)^{n+1} 1!$. Use a computer in order to find the smallest number n such that a_n is a composite number.

(148) The mathematicians Minác and Willans have obtained a formula for the n-th prime number p_n which is more of a theoretical interest than of a