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Worksheet 8: Nonlinear Systems of Differential Equations

Nonlinear Systems of Differential Equations—Consumer-Resource Models

Nonlinear, autonomous systems of ordinary differential equations are of the form

$$\begin{aligned}\frac{dx_1}{dt} &= f_1(x_1, x_2, \ldots, x_n)\\ \frac{dx_2}{dt} &= f_2(x_1, x_2, \ldots, x_n)\\ &\vdots\\ \frac{dx_n}{dt} &= f_n(x_1, x_2, \ldots, x_n)\end{aligned}$$

where each of the functions f_i on the right-hand side are real-valued functions in n variables. Most of the time, we will restrict the analysis to systems of two variables. We will focus on equilibria and stability.

Equilibria and Stability

Consider the system of two autonomous differential equations

$$(1)\qquad \begin{aligned}\frac{dx}{dt} &= f(x, y)\\ \frac{dy}{dt} &= g(x, y)\end{aligned}$$

The first step is to find the equations of the zero isoclines, which are defined as the set of points that satisfy

$$\begin{aligned}0 &= f(x, y)\\ 0 &= g(x, y)\end{aligned}$$

Each equation results in a curve in the x-y space. Point equilibria occur where the two isoclines intersect (Figure 1). A point equilibrium $(\hat{x}, \hat{y})$ of (1) therefore simultaneously satisfies the two equations

$$f(\hat{x}, \hat{y}) = 0 \quad \text{and} \quad g(\hat{x}, \hat{y}) = 0$$

We will call point equilibria simply "equilibria."